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# Development of a mathematics teaching module based on the APOS model integrated with GeoGebra

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**Abstract.** This study aims to develop a mathematics teaching module based on the APOS (Action, Process, Object, Schema) model integrated with GeoGebra, focusing on the topic of quadratic functions for tenth-grade students at SMAN 1 Muara Beliti. The research employed the Plomp development model, which consists of five phases: preliminary investigation, design, construction, validation, and limited testing. The module was developed under the cognitive stages of the APOS model: action, process, object, and schema, and was enhanced through visual exploration using GeoGebra. The research subjects consisted of tenth-grade students and a mathematics teacher at SMAN 1 Muara Beliti, selected through purposive sampling. Data were collected through documentation, expert validation sheets, and practicality questionnaires completed by the teacher and students. The findings indicate that the developed module is highly needed by students and significantly addresses their difficulties in understanding mathematical concepts. Expert validation yielded an average score of 87.85%, classified as highly valid. Practicality testing produced scores of 92.50% from the teacher and 90.41% from the students, both categorized as highly practical. Thus, the module facilitates interactive, conceptual, and contextual learning aligned with the principles of the Independent Curriculum. Accordingly, it is deemed a feasible and innovative instructional resource for high school mathematics learning.

## Introduction

In line with the rapid advancement of science and technology, which significantly influences the field of education, improving the quality of education has become one of the government's top priorities. To support this effort, teachers are encouraged to utilize learning resources beyond those available at school in order to enhance the teaching and learning process. The integration of technology into education is commonly implemented through the use of computers. By using educational applications, students can understand the subject matter more easily, as they can directly observe displayed visuals and independently revisit the material that they have not yet fully understood (Asdarina & Khatimah, 2021).

Education is a fundamental aspect of human life that plays a vital role in improving the quality of life. It not only helps individuals understand various concepts and bodies of knowledge but also drives change and innovation across multiple areas of life. Through quality education, individuals can develop critical thinking skills, analytical abilities, and an innovative mindset that are essential in today's era of globalization. Mathematics is one of the core subjects that holds a crucial role in education (Nuraini & Afifurrahman, 2023; Widana et al., 2024). Quadratic functions are also relevant to everyday life, such as in modeling the trajectory of objects, financial planning, and calculating maximum or minimum areas. Therefore, mastering the concept of quadratic functions comprehensively is of great importance (Arifin et al., 2024).

Based on field observations conducted at SMAN 1 Muara Beliti, particularly in Grade X, many students continue to experience difficulties in comprehensively understanding mathematical concepts, especially on abstract topics such as quadratic functions. Quadratic functions are a critical topic that requires students to grasp algebraic forms, graphical representations, and the relationships among coefficients in quadratic equations. Many students tend to memorize formulas without understanding the underlying concepts, which often leads to misconceptions when they are asked to interpret parabolic graphs or solve context-based problems. One common mistake is in graphing quadratic functions. Numerous students draw graphs without considering proportional scaling, accurate axes of symmetry, or the correct curvature of the parabola, often producing sharp or angular representations. Some students even sketch the parabola solely based on the vertex, ignoring the direction of the opening or the values of the coefficients. These findings indicate that students have not yet developed a clear geometric and visual understanding of quadratic functions.

In addition, many students perceive mathematics as a complicated and intimidating subject. As a result, their interest in learning mathematics tends to be low, which leads to difficulties in understanding the concepts being taught. The primary goal of mathematics instruction is to enable students to understand mathematical concepts, develop logical thinking patterns, and apply reasoning in problem solving (Nuraini & Afifurrahman, 2023). These difficulties often stem from a heavily procedural teaching approach. The textbooks used by teachers generally present problem-solving steps directly without guiding students first to build a conceptual understanding (Herdian et al., 2019; Sumandya et al., 2023). Most students follow procedures by substituting numbers into formulas without grasping the symbolic meaning or geometric representation of quadratic functions. As a result, they demonstrate weak problem-solving skills and a limited ability to connect different forms of representation, such as symbols, graphs, and narratives (Sukendra et al., 2023).

Unfortunately, various studies have shown that students' understanding of quadratic functions remains low. According to Sari and Hoiriyah (2021), one of the most common errors made by students is in graphing parabolas and identifying their vertices. This is supported by field findings at SMAN 1 Muara Beliti, which revealed that more than 60% of students drew quadratic function graphs not as smooth curves, but rather as straight lines or sharp, angular shapes, without considering the axis of symmetry or the direction in which the parabola opens. Students also struggled to associate the coefficients a, b, and c with the shape of the parabola such as whether it opens upward or downward, or whether the graph is wide or narrow. In response to these issues, I intend to develop a mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions, in order to help students gain a more comprehensive conceptual understanding and to produce an innovative learning material that is both valid and practical for classroom use.

This challenge becomes even more complex when viewed in light of the characteristics of today's learners, Generation Z, who are highly familiar with digital technology yet tend to have shorter attention spans and a preference for active, visual, and instant forms of learning. As such, instructional material innovation has become an urgent necessity. One relevant approach is the use of the APOS model (Action, Process, Object, Schema), a constructivist framework grounded in the stages of mathematical thinking. This model has been shown to enhance students' conceptual and reflective thinking abilities (Antasari et al., 2023; Purnadewi & Widana, 2023).

A teaching module is a component of instructional tools used by educators as a reference for implementing the learning process, intending to achieve the Pancasila Student Profile (P3) and the expected learning outcomes. Teachers play a central role in the development of teaching modules, as they best understand the subject matter to be taught, the intended learning objectives, and the characteristics of their students. The primary goal of developing a teaching module is to produce instructional materials that serve as a guide for teachers in conducting lessons that meet specific criteria aligned with students' needs and learning profiles (Rusmini et al., 2023; Pratama et al., 2024).

To make this approach more effective, support from interactive and visual learning media is essential. GeoGebra is one such application that is highly suitable for helping students dynamically explore the concept of quadratic functions. GeoGebra allows learners to manipulate function parameters and directly observe changes in the graph's shape, thereby strengthening the connection between algebraic forms and graphical representations. According to Siregar (2017), the use of GeoGebra enhances students' conceptual understanding and fosters curiosity as well as active engagement in learning mathematics. The Independent Curriculum emphasizes the importance of meaningful and differentiated learning that supports the development of the Pancasila Student Profile (Anto et al., 2024).

Therefore, this study aims to develop a valid and practical mathematics teaching module based on the APOS model integrated with GeoGebra for teaching quadratic functions in tenth-grade high school. The module is intended to serve as an alternative, innovative learning tool to support meaningful and interactive instruction in the context of the Independent Curriculum era (Dini et al., 2020).

The APOS model posits that an individual's mathematical knowledge and understanding reflect their tendency to respond to a mathematical situation and interpret it within a social context. Through this process, individuals construct or reconstruct mathematical ideas via actions, processes, and objects, which are then organized into a scheme that can be applied to solve problems. A scheme in mathematical concepts consists of a collection of actions, processes, objects, and other interconnected schemes related to a particular concept (Budiarti et al., 2019). According to Dubinsky, the APOS theory explains how a child's mental activities, namely action, process, object, and schema, are involved in constructing mathematical concepts. A child can effectively build mathematical understanding when they experience all four components: action, process, object, and schema (Syam, 2021).

The APOS model is considered highly suitable for optimizing students' understanding, as it has been widely applied to stimulate students in constructing their knowledge through its framework of action, process, object, and schema. Constructing and reconstructing mathematical objects through these stages is a reflection of students' conceptual understanding in mathematics (Putri et al., 2022).

The research problems addressed in this study are as follows: (a) How is the development process of a mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions for tenth-grade high school students? (b) What is the level of validity of the mathematics teaching module based on the APOS model integrated with GeoGebra, as assessed by experts? (c) What is the level of practicality of the mathematics teaching module based on the APOS model integrated with GeoGebra, as evaluated through limited trials involving teachers and tenth-grade students? The objectives of this study are as follows: (a) To develop a mathematics teaching module based on the APOS model integrated with the GeoGebra application. (b) To determine the validity level of the developed mathematics teaching module based on the APOS model integrated with GeoGebra, as assessed by experts. (c) To determine the practicality level of the developed teaching module based on the results of limited trials conducted by teachers and tenth-grade students.

## Method

This study was conducted at SMAN 1 Muara Beliti, Musi Rawas Regency, South Sumatra Province. The research subjects were selected using purposive sampling. Accordingly, the participants consisted of tenth-grade students and a tenth-grade mathematics teacher at SMAN 1 Muara Beliti. This study is a development research aimed at producing a valid and practical mathematics teaching module based on the APOS model integrated with GeoGebra, to be used in teaching quadratic functions in Grade 10. The resulting product is a mathematics teaching module based on the APOS model integrated with GeoGebra, available in both print and digital formats.

The development model used in this study is the Plomp model, which consists of several phases. The Plomp model includes five main phases: Preliminary Investigation, Design, Realization/Construction, Testing, Evaluation and Revision, and Implementation (Hidayat et al., 2022). The stages of the Plomp development model are as follows:

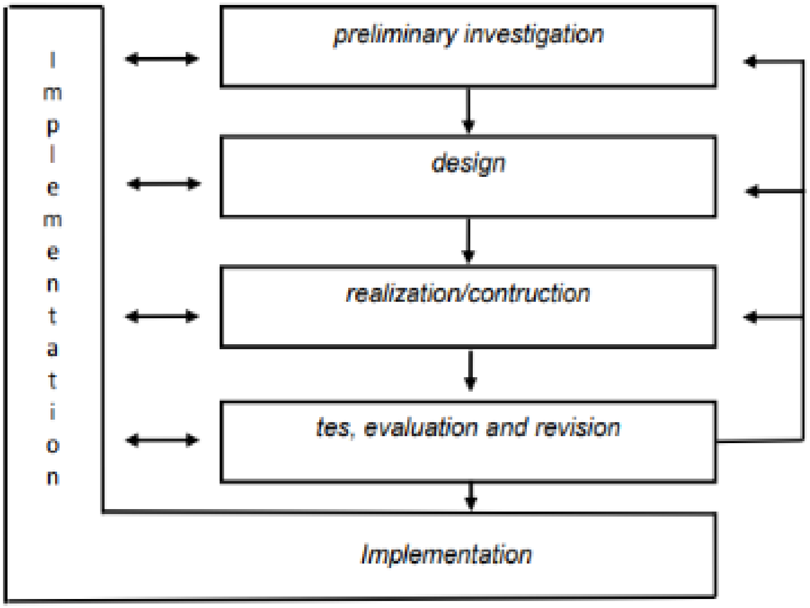


**Image 1.** Plomp Model

Additional details regarding the steps of the validity testing are as follows:

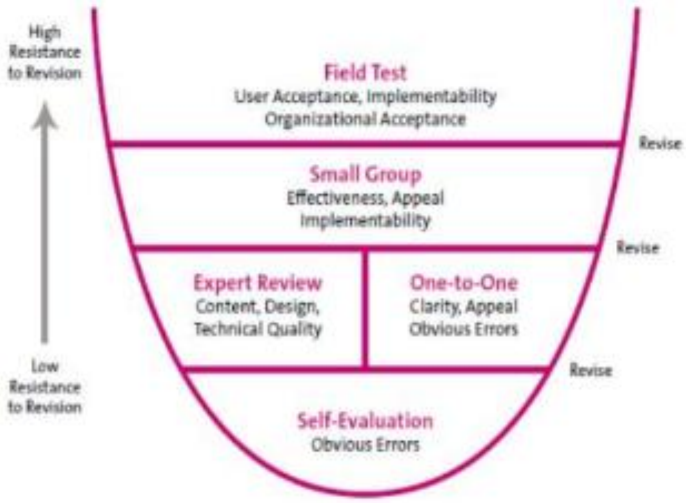


**Image 2.** Layers of Formative Evaluation in the Plomp Development Model

Image 2 illustrates that the development process begins with self-evaluation, followed by revisions. The product is then submitted to experts for evaluation of its content, design, and overall quality. In addition, it is tested with individual users in a one-on-one setting. Feedback and recommendations from both experts and individual users are collected for further revision. Once revised, the product is re-evaluated, and if deemed valid, it proceeds to small-group testing. After the small-group trial, another evaluation is conducted to determine whether further revisions are necessary. Only after these stages is the product tested in a field trial.

The data collection techniques used in this study consisted of teacher reflections and questionnaires. The data analysis techniques involved two approaches: qualitative descriptive analysis and quantitative descriptive analysis. Both methods were employed to evaluate the validity and practicality of the mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions. The analysis was conducted based on feedback from experts, teachers, and students. For expert analysis, a validity test was conducted using a qualitative validation approach, and the data were analyzed using mean score calculations. For the practicality test, data were obtained from questionnaires completed by teachers and students who directly used the teaching module and student worksheets (LKPD). Student responses were analyzed using a Likert scale ranging from 1 to 4 regarding their experience with the developed module and LKPD during the learning process. This technique allowed the researcher to assess the degree of validity and practicality of the developed product and to determine whether the product was ready for further testing or required revision (Sugiyono, 2019).

## Results and Discussion

The instructional material developed in this study is a mathematics teaching module based on the APOS model (Action, Process, Object, Schema) integrated with the GeoGebra application. Accordingly, the validation of the module specifically focused on key aspects that support constructivist-based learning and interactive visualization. These aspects include the coherence of APOS stages, the appropriateness of the content, the use of communicative language, the visual presentation, and the effectiveness of GeoGebra integration as a learning aid.

The validation data were analyzed using descriptive quantitative methods to determine the level of acceptability (validity) of the developed module. This study focused its evaluation on the teaching module as the primary instructional material designed to support the learning of quadratic functions. Prior to implementation, the module was validated by a subject matter expert, a media expert, and a language expert. The purpose of this validation was to obtain feedback and suggestions regarding the content relevance, clarity of presentation structure, integration of GeoGebra, and alignment with student characteristics, in order to ensure that the module is ready for use in the learning process.

The validity of the teaching material was assessed by experts who provided scores for each aspect specified in the validation instrument. The scores were then converted into percentages to determine the validity level of each aspect of the module. This study employed the Plomp development model to produce a mathematics teaching module on the topic of quadratic functions. The Plomp model consists of five stages: preliminary investigation, design, realization/construction, test, evaluation and revision, and limited implementation.

**Preliminary Investigation Phase**

The preliminary investigation phase was conducted to thoroughly identify the need for developing a contextual teaching module on quadratic functions that aligns with real conditions in the field. One of the key aspects examined was curriculum analysis, in which the researcher reviewed the Learning Outcomes (*Capaian Pembelajaran*, or CP) for Phase E of the Independent Curriculum. This curriculum emphasizes the importance of conceptual understanding, reasoning skills, and the integration of technology in mathematics instruction, including in the topic of quadratic functions.

Based on interviews with the mathematics teacher at SMAN 1 Muara Beliti, it was found that students' initial ability to understand quadratic functions varied widely. Some students continued to struggle with identifying the general form of a quadratic function and were unable to draw parabolic graphs accurately. Common errors included graph representations resembling straight lines, asymmetric curves, or incorrect directions of the parabola's opening. These conditions indicate a weak understanding of the relationship between the coefficients in a quadratic equation and the corresponding visual shape of its graph.

The teacher also noted that there is currently no teaching module available that systematically bridges the gap between symbolic and visual representations, making it difficult for students to construct a comprehensive mental model of the quadratic function concept (Marfuah et al., 2023). The textbook in use is procedural and lacks exploratory activities, leading students to merely solve problems without engaging in deeper conceptual understanding through visualization or reflection on mathematical meaning. Instructional technologies such as GeoGebra have not been utilized optimally, despite their strong potential to support dynamic and interactive visual learning (Widana & Laksitasari, 2023).

The teacher also reported that students often exhibit signs of disengagement during lessons due to monotonous and insufficiently challenging instructional methods. These findings underscore the urgency of designing a teaching module based on the APOS model that can facilitate the gradual development of students' cognitive structures. Additionally, the integration of GeoGebra as an interactive visualization tool is essential to effectively and meaningfully bridge symbolic and graphical representations (Martina et al., 2015).

**Design & Construction**

Based on the results of the preliminary investigation, the researcher developed an initial draft of the teaching module with structure and content aligned to the APOS model approach and the integration of GeoGebra. Several key activities carried out during this phase included: 1) Module Structure Design: The module was organized with the following key components: module identity, learning objectives, instructional content, student activities based on the APOS framework (action, process, object, schema), instructions for using GeoGebra, formative assessments, and reflective learning prompts. 2) Learning Activity Design: The learning activities in the module were structured progressively following the APOS stages. In the action stage, students are given simple exploratory tasks; in the process stage, they begin to formulate generalizations; in the object stage, students draw conceptual conclusions; and in the schema stage, they apply the concepts in new contexts. 3) Integration of GeoGebra Media: The module guides users using GeoGebra to visualize quadratic function graphs dynamically. GeoGebra is utilized in student exploration activities to allow them to observe how changes in coefficient values affect the shape of the parabola visually. 4) Prior Competencies: Identifying the general form of a quadratic function, understanding the roles of coefficients a, b, and c, and recognizing the parabolic graph as a representation of a quadratic function. 5) Pancasila Student Profile: Faith in God Almighty and noble character; independence; critical thinking; creativity; and collaboration. 6) Learning Objectives (LO): Through the APOS learning model and the use of the GeoGebra application, students are expected to understand how the values of a, b, and c affect the shape of a quadratic function graph, as well as accurately present and interpret the graph of a quadratic function. 7) Learning Objective Pathway (ATP): Students can explain the general form of a quadratic function; identify the influence of coefficients on the direction and width of the parabola; construct quadratic function graphs using GeoGebra; and interpret the graph of a quadratic function in contextual problems.

**Construction/Realization Phase**

The construction phase refers to the process of developing the teaching module based on the initial design formulated in the previous stage. The product developed in this study is a mathematics teaching module based on the APOS model integrated with GeoGebra for teaching quadratic functions in tenth-grade high school. The developed product includes: a) Module Format: The module was designed in accordance with the standard format of the Independent Curriculum, which includes module identity, learning objectives, concept map, learning activities, reflection, assessment, and follow-up actions. b) Structure of Learning Activities: Learning activities were structured based on the stages of the APOS model: 1) Action: Students are given sample problems to trigger initial actions in understanding quadratic functions, 2) Process: Students explore relationships among function components through observation and discussion, 3) Object: Students begin to recognize quadratic functions as mathematical objects that can be manipulated. 4) Schema: Students connect the understanding they have built into a coherent mental schema. a) GeoGebra Integration: Each activity in the module is accompanied by links or QR codes to interactive GeoGebra files. Students are instructed to manipulate the values of a, b, and c to observe how the shape of the quadratic function graph changes. This feature aims to strengthen the visual connection between algebraic forms and parabolic graphs. b) Development of Student Activity Sheets (LKPD): The module also includes student worksheets (LKPD) designed to guide learners through a gradual and independent exploration of quadratic functions. The LKPD includes: 1) GeoGebra exploration guides, 2) Guiding questions, 3) Concept reinforcement tasks, 4) Reflection and self-feedback sections. a) Language and Visual Design Adjustments: The language used in the module is tailored to the students' developmental level and follows the rules of the Indonesian Language Spelling System (PUEBI). The module layout is visually engaging, incorporating icons, colors, and illustrations that support readability and enhance students' motivation to learn. b) Accessibility and Implementation Feasibility: The module is designed in a digital format (PDF) that allows students to access it online. It can also be printed for use in offline learning environments. This design ensures flexibility and adaptability to various school learning conditions.

**Evaluation and Revision Phase**

After the initial draft of the module was completed, an evaluation process was carried out through expert validation and limited practicality testing. This process included: 1) Expert Validation: The module was evaluated by three expert validators in the areas of content, language, and media. The assessment was conducted using a Likert-scale validation sheet, accompanied by comments and suggestions for improvement. 2) Module Revision: The module was revised based on the feedback provided by the validators to improve content accuracy, activity presentation, language clarity, and visual design. 3) Practicality Testing: The module was tested on a limited basis by one mathematics teacher and six tenth-grade students. They assessed the module in terms of ease of use, readability, usefulness, and content integration.

The evaluation results indicated that the module met the criteria for validity and practicality and is considered suitable for use in teaching quadratic functions in tenth-grade high school mathematics.

**Validation Results**

The validation in this study aimed to assess the feasibility of the mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions before it was tested on students. The validation was carried out by three expert validators, each with relevant expertise in their respective fields.

**Language Expert Validation**

Drs conducted language validation. Hendra Wadi, M.Pd. (Principal School Supervisor at the South Sumatra Provincial Education Office). Overall, the validation results indicated that the language used in the teaching module was appropriate and communicative. The sentences were practical and easy for students to understand, and the terminology was relevant to the mathematics content and GeoGebra application. However, several suggestions for improvement were noted, particularly regarding consistency in the use of mathematical terms and symbols, as well as the simplification of specific sentence structures to enhance student accessibility.

**Content Expert Validation**

The content validation aimed to evaluate the accuracy and appropriateness of the subject matter in the mathematics teaching module, based on the APOS model integrated with GeoGebra, specifically on the topic of quadratic functions. Saidina Umar, M.Pd.Mat conducted this validation. (Principal of SMA Negeri Marga Baru). The validator assessed the module based on indicators such as content quality, conceptual correctness, integration of the APOS model and technology, development of student worksheets (LKPD), and instructional feasibility.

**Media Expert Validation**

The media validation was conducted to evaluate the extent to which the mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions fulfilled the criteria for visual feasibility, readability, presentation consistency, and the integration of technology as a learning medium. Vera Hersanti, M.Pd, carried out this validation. (Teacher at SMA Negeri 1 Pagar Alam). The assessment covered six leading indicators: (1) module dimensions, (2) visual appearance, (3) font usage and formatting, (4) language accuracy within the media, (5) layout consistency, and (6) integration of the APOS model and GeoGebra as visual aids to support the understanding of quadratic function concepts. Each indicator comprised several items rated on a 4-point Likert scale (1 = Poor, 2 = Fair, 3 = Good, 4 = Excellent).

Based on the validation results conducted by the three expert validators for the mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions, assessment data were obtained across four key aspects: content and presentation of the teaching module, language use, accuracy of the material, and visual appearance and integration of instructional media. Each aspect was evaluated by the respective expert according to their field of specialization, with scores assigned based on predetermined assessment indicators.

The validation results indicated that the teaching module received excellent ratings across all evaluated aspects. For content and module structure, the average score fell into the "very good" category at 86.65%. The language aspect received an average score of 89.28%, classified as "excellent." The material aspect also achieved an "excellent" rating with an average score of 85.55%. Meanwhile, for the instructional media aspect, the average score was 89.93%, also falling into the "very good" category.

These findings indicate that the teaching module has met the feasibility standards in terms of content, language, media, and material. Overall, the combined average score across all validated aspects was high, suggesting that the module is suitable for use in tenth-grade high school mathematics instruction, with only minor refinements needed in the areas of visual media and the variety of exploratory questions.

Based on feedback from the validators, several substantive and technical revisions were made. These improvements aimed to enhance the quality of the teaching module, making it more systematic, communicative, and supportive of the achievement of learning objectives in accordance with the Independent Curriculum. A comparison between the pre-validation and post-validation versions of the teaching module is presented in the following table:

**Table 1.** Comparison of the Teaching Module Before and After Validation

| Aspect | Before Validation | After Validation |
|---|---|---|
| Learning Objectives | Not fully explicit in the ABCD formulation. | Clarified by explicitly stating the audience, behavior, condition, and degree. |
| Content Relevance | Relevant, but contextual questions were still limited. | Enriched with real-world problem-based exploration tasks and GeoGebra-supported enrichment items. |
| GeoGebra Integration | Integrated, but lacked systematic technical instructions. | Practical GeoGebra steps were added in both textual and illustrated formats. |
| Language and Spelling | Some sentences were lengthy and ineffective. | Simplified for better clarity and communicative style, aligned with PUEBI (Indonesian language standards). |
| Mathematical Terminology | Some terms, such as "axis of symmetry" and "vertex form," were used inconsistently. | Terminology consistency was improved throughout the module, including in the LKPD and assessment items. |
| Glossary | Did not fully cover all technical terms. | Expanded to include key mathematical terms that support student understanding. |

| Aspect | Before Validation | After Validation |
|---|---|---|
| Layout and Media | Layout was not entirely proportional; illustrations were limited. | Design was adjusted to be more engaging, with better use of colors and balanced graphical elements. |
| Learning Reflection | Only included as a final task. | Added guided reflective questions for both teachers and students. |
| Assessment Appendices | Included only LKPD and practice questions. | Added assessment rubrics, remedial worksheets, and enrichment tasks supported by GeoGebra. |

Overall, the teaching module demonstrated significant improvements in terms of presentation, technological integration, and the APOS-based scientific approach. The module became more communicative, interactive, and applicable in supporting the achievement of learning outcomes and the development of the Pancasila Student Profile. Based on validation across four key aspects: content expert validation, language expert validation, media expert validation, and overall module validation, it can be concluded that the mathematics teaching module based on the APOS model integrated with GeoGebra on the topic of quadratic functions is deemed suitable for use in tenth-grade high school mathematics instruction.

**Practicality Test Results**

The practicality test was conducted after the teaching module was validated by subject matter experts, language experts, media experts, and teaching module experts. This test aimed to determine the extent to which the APOS model-based mathematics teaching module integrated with GeoGebra could be used practically in learning activities. Practicality was measured based on aspects such as ease of use, clarity of instructions, activity flow, language appropriateness, and the visual presentation of media that supports the teaching and learning process.

Following the validation process by experts, the APOS model-based mathematics teaching module integrated with the GeoGebra application was subsequently tested to determine its level of practicality. This practicality test aimed to evaluate how effectively and efficiently the module could be used by practitioners in real classroom settings. The practicality assessment conducted by teachers aimed to gather information on the ease of use, clarity of presentation, the module's usefulness in supporting learning, and the extent to which the APOS learning model and GeoGebra application integration was reflected in teaching and learning activities. The results of the module's practicality assessment by teachers are presented in the following table:

**Table 2.** Teacher Assessment of Practicality

| No | Assessment Indicator | Assessment Statement | Score |
|---|---|---|---|
| 1 | Ease of Use | The module is easy to use in learning | 4 |
| | | Activity instructions are easy to understand | 4 |
| | | GeoGebra practicum runs systematically | 4 |
| 2 | Presentation | The design is attractive and consistent | 4 |
| | | The language used in the module is easy for students to understand | 4 |
| | | The module layout is neat and professional | 3 |

| No | Assessment Indicator | Assessment Statement | Score |
|---|---|---|---|
| 3 | Usefulness | The module encourages students' independent learning activities | 4 |
| | | The module enhances conceptual understanding of quadratic functions visually | 4 |
| 4 | Integrasi APOS & GeoGebra | The module integrates APOS phases with exploratory practice through GeoGebra | 3 |
| | | GeoGebra activities strengthen student analysis | 3 |
| Total Score | | | 37 |
| Maximum Score (10 × 4) | | | 40 |

After obtaining validation from experts and a practicality test by the teacher, the next step was to conduct a practicality test involving students. The purpose of this activity was to determine the extent to which the APOS model-based mathematics teaching module integrated with GeoGebra, specifically on the topic of quadratic functions, could be practically used by students during the learning process. The practicality assessment by students focused on ease of understanding the module content, visual appearance, clarity of instructions, exploratory activities using GeoGebra, and the module's impact on students' motivation and confidence in learning mathematics.

This practicality test was conducted with six Grade X students from SMA Negeri 1 Muara Beliti, selected to represent diverse student characteristics and abilities. The instrument used was a closed-ended questionnaire employing a Likert scale. Each student evaluated ten statements designed according to practicality indicators, namely: ease of use, presentation, usefulness, and integration of the APOS learning model with the GeoGebra application. The following are the results of the practicality assessment by students.

**Table 3.** Student Practicality Assessment

| Student | Maximum Score | Score Obtained | Percentage (%) | Category |
|---|---|---|---|---|
| S1 | 40 | 35 | 87.50% | Very Practical |
| S2 | 40 | 38 | 95.00% | Very Practical |
| S3 | 40 | 36 | 90.00% | Very Practical |
| S4 | 40 | 36 | 90.00% | Very Practical |
| S5 | 40 | 37 | 92.50% | Very Practical |
| S6 | 40 | 35 | 87.50% | Very Practical |
| Average | – | – | 90.41% | Very Practical |

Based on the results of the practicality test conducted on the APOS model-based mathematics teaching module integrated with GeoGebra for the topic of quadratic functions, it can be concluded that the module meets the criteria as a practical and feasible teaching material for use in Grade X classroom learning.

The practicality test was carried out on two user groups: teachers as practitioners and students as direct users in the classroom. Each group provided assessments using a Likert scale questionnaire covering several indicators, including ease of use, appearance, clarity of instructions, usefulness, and the integration of the learning model with technology.

The practicality results from teachers showed a percentage of 92.50%, which falls into the Very Practical category. Teachers assessed that the module effectively facilitates exploration-based learning, features an attractive layout, and provides sufficiently clear instructions for using GeoGebra. However, it was suggested that the module could be improved by reinforcing the exploratory steps and adding a more systematic reflection section at the end of the activities.

The practicality results, based on six respondents, indicated an average percentage of 90.41%, also falling under the Very Practical category. Students felt that the module helped them understand the topic of quadratic functions visually and interactively. The module was considered easy to understand, visually appealing, and the GeoGebra activities were seen as stimulating curiosity and encouraging active engagement in learning.

Thus, the overall results of the practicality tests, both from teachers and students, indicate that this teaching module is efficient for use in high school mathematics learning. The module not only supports conceptual understanding through the APOS approach and GeoGebra-based exploration, but is also systematically and communicatively structured, thereby supporting both independent and collaborative learning..

The research findings indicate that the APOS model-based mathematics teaching module, integrated with GeoGebra, which was developed, is valid and highly practical for use in teaching quadratic functions in Grade X of senior high school. The average validation result reached 87.85%, while the practicality score was 90.41%, both falling into the very good to convenient categories. This study aligns with the findings of Simorangkir (2023), which showed that the validation questionnaire results for the material aspect reached 80%, and for the design aspect, 91.11%. The teacher's practicality percentage was 91.11%, while the students' practicality was 93.77%. The effectiveness level based on the student effectiveness questionnaire reached 95.11%. The average student learning outcome test score was 81%, categorized as very effective. Meanwhile, the average score of the student motivation questionnaire after using the M-APOS model-based e-module integrated with an animated film via Toontastic 3D was 89.46%, categorized as high. These results suggest that the M-APOS model-based e-module integrated with animated film (Toontastic 3D) can significantly enhance students' motivation to learn mathematics.

Teacher and student validations indicate that the structure of this module is highly practical, both in terms of technical usability and the meaningfulness of the activities. The module provides exploratory instructions, practicum steps, and conceptual reflection that foster students' metacognitive skills. This aligns with the demands of 21st-century learning, which emphasizes the ability to learn independently, think critically, and make informed decisions. The module also includes remedial and enrichment features based on GeoGebra activities, which align with the principles of differentiated learning as promoted in the Kurikulum Merdeka. This means that students with varying abilities are fairly and proportionally accommodated. These findings reinforce the results of a study by Marsitin (2018), which stated that instructional tools designed with constructivist principles and digital media can enhance students' learning independence and participation. Meanwhile, Anggraeni et al. (2021) noted that dynamic visualizations through GeoGebra contribute significantly to overcoming misconceptions in quadratic function graphs, such as incorrect scaling or misrepresentation of the parabola's shape.

Conceptually, the content validity of the module is also evident in the selection of quadratic function material, which is presented through visual concretization and real-life contexts. This is crucial, as many students experience misconceptions when drawing quadratic graphs, for example, rendering the parabola rigidly or with incorrect scaling (Tilari et al., 2024). GeoGebra has proven effective in providing immediate feedback, allowing students to revise and improve their understanding reflectively.

The results of this study present a novel contribution to the development of a mathematics teaching module that integratively combines the APOS instructional model with the digital GeoGebra application for the topic of quadratic functions. Unlike previous studies that tended to apply the APOS model and GeoGebra separately, this research systematically designed an instructional tool based on the stages of APOS thinking (Action, Process, Object, Schema), facilitated through the interactive visualizations offered by GeoGebra. This module was not designed merely as a conventional teaching resource, but as a means for deep, technology-based mathematical concept construction aligned with the principles of the Independent Curriculum. The validation conducted by experts, along with practicality testing by teachers and students, yielded ratings of high validity and high practicality, confirming that this module is a viable instructional innovation that has not yet been widely developed in a similar form.

However, this study has several limitations that should be considered when interpreting the results. First, the development of the mathematics teaching module based on the APOS model integrated with GeoGebra was carried out only up to the stages of validity and practicality testing; therefore, the module's effectiveness in improving student learning outcomes has not yet been quantitatively measured. Second, the limited trial involved only one teacher and six students from a single school, which restricts the generalizability of the findings to that specific context. Third, the use of GeoGebra within the module requires access to adequate technological devices such as laptops or mobile devices, which may present implementation challenges in schools with limited ICT facilities. These limitations form the basis for future research to evaluate the module's effectiveness more broadly and to develop implementation strategies that are adaptable to varying infrastructure conditions.

The results of this study have significant theoretical and practical implications in the field of mathematics education. Theoretically, this research enriches the scholarly discourse on the development of instructional materials based on the APOS model with the integration of digital media, specifically GeoGebra an area that has rarely been fully developed in the form of a comprehensive teaching module. This study reinforces the relevance of the constructivist approach in mathematics learning, particularly in fostering conceptual understanding through systematic stages of thinking. Practically, the developed teaching module can be directly utilized by teachers and students in the classroom as an interactive, visual, and self-directed learning resource. The module also aligns with the principles of the Independent Curriculum, as it supports differentiated instruction, technology integration, and the reinforcement of the Pancasila Student Profile. With its high levels of validity and practicality, this module holds strong potential as a concrete solution to help students overcome difficulties in understanding quadratic functions both visually and conceptually in tenth-grade mathematics.

## Conclusion

This study aimed to develop a mathematics teaching module based on the APOS model integrated with GeoGebra for the topic of quadratic functions in Grade X of senior high school. Based on the development process, expert validation, and practicality testing, several conclusions can be drawn: The developed teaching module was declared valid based on validation results from three experts assessing the aspects of structure and content, language, subject matter, and media, with an average score of 87.85%, which falls into the "good" category. The module met the eligibility criteria in terms of structural organization, content alignment with the Independent Curriculum, use of communicative and grammatically correct language (in accordance with PUEBI), and consistent and visually appealing media design. The APOS model integrated with GeoGebra was also considered suitable for the characteristics of the quadratic function material and the needs of 21st-century learners. Furthermore, the module was found to be highly practical for use by teachers and students, as evidenced by an average practicality score of 90.41%. The module is easy to use, facilitates systematic learning activities, and supports differentiated instruction.

Teachers reported that the module could be immediately implemented within the learning scenarios of the Independent Curriculum. At the same time, students found the dynamic visualizations using GeoGebra helpful for understanding the graph of quadratic functions. The teaching module successfully integrates a conceptual approach (APOS) with dynamic visual media (GeoGebra) into a cohesive instructional tool. This integration fosters a learning process that promotes active engagement, conceptual reflection, and exploration of mathematical meaning, ultimately helping to address students' misconceptions regarding quadratic function graphs, including issues related to scale, the shape of the parabola, and the vertex. The module has demonstrated its contribution to the development of innovative instructional materials aligned with the principles of the Independent Curriculum and the reinforcement of the Pancasila Student Profile, through the integration of technology, meaningful learning experiences, and the facilitation of differentiated instruction.

Based on the findings of this study, several recommendations can be made. Teachers are encouraged to adapt the module according to their school context and the technological readiness of their students, as well as to guide in using GeoGebra during the initial stages of implementation. Future researchers are advised to extend this study to the effectiveness-testing phase in order to quantitatively assess the impact of the module on student learning outcomes, conceptual understanding, and critical thinking skills.

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