



## Identifying key factors influencing mathematical visual thinking among pre-service mathematics teachers

Ria Noviana Agus<sup>\*)1</sup>, Rina Oktaviyanthi<sup>2</sup>

<sup>1</sup>Universitas Serang Raya, Serang, Indonesia; [riaagus@unsera.ac.id](mailto:riaagus@unsera.ac.id)

<sup>2</sup>Universitas Serang Raya, Serang, Indonesia; [rinaokta@unsera.ac.id](mailto:rinaokta@unsera.ac.id)

<sup>\*)</sup>Corresponding author: Ria Noviana Agus; E-mail addresses: [riaagus@unsera.ac.id](mailto:riaagus@unsera.ac.id)

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**Abstract.** Mathematical visual thinking is essential for understanding complex mathematical concepts, as it enables learners to interpret and connect visual and symbolic representations. However, many pre-service mathematics teachers struggle to integrate visual representations with algebraic expressions, leading to conceptual misunderstandings. This study aims to identify the key factors influencing mathematical visual thinking among pre-service mathematics teachers. A mixed-methods descriptive study was conducted, involving 150 students for quantitative analysis and 15 selected participants for qualitative interviews. Data were collected through a mathematical visual thinking test, a visual thinking factors questionnaire, semi-structured interviews, and document analysis of students' mathematical tasks. The results indicate that most students exhibit moderate visual thinking ability (50%), with challenges in linking graphical and algebraic representations. The key influencing factors include teaching

approaches (4.3), concept understanding (4.1), motivation (4.0), and technology use (4.2), whereas anxiety plays a lesser role (2.9). Students relying heavily on technological tools tend to improve accuracy but struggle with independent visualization. These findings suggest that instructional strategies should integrate structured visual exercises, emphasize transformation understanding, and balance technological support with manual problem-solving. This study provides new insights into enhancing visual thinking in mathematics and offers pedagogical recommendations for pre-service teacher education.

## Introduction

Mathematical visual thinking is a crucial ability that enables individuals to understand, interpret, and communicate mathematical ideas through visual forms such as graphs, diagrams, spatial reasoning, and dynamic images (Agus & Oktaviyanthi, 2023; Arnheim, 2020; Elsayed & Al-Najrani, 2021; Dwikamayuda, 2024). This skill facilitates deeper understanding of abstract structures, promotes cognitive flexibility, and enhances problem-solving effectiveness (Geçici & Türnükü, 2021; Svitek et al., 2022). In the context of pre-service mathematics teacher education, this ability becomes even more vital, as future educators must visually convey concepts to improve student understanding (Nutov, 2021; Zulu & Mudaly, 2023). Research indicates that teachers with strong visual thinking abilities are more effective in conveying abstract mathematical concepts in an intuitive manner (Vale & Barbosa, 2023; Zulu & Mudaly, 2023; Panduwinata et al., 2025). Moreover, visual thinking accelerates problem-solving processes and enhances flexibility in

approaching multiple solution strategies (Dunstan & Cole, 2021; Kim & Park, 2021; Yeh et al., 2024). Therefore, pre-service mathematics teachers must develop optimal visual thinking skills to translate mathematical concepts into accessible and meaningful representations for students (van Garderen et al., 2021; Wilkie, 2021; Widana & Ratnaya, 2021).

However, several studies indicate that many pre-service mathematics teachers face persistent challenges in using visual thinking effectively (Coessens et al., 2021; Mainali, 2021). These difficulties include poor interpretation of graphs, inaccuracies in drawing geometric objects, and weak connections between visual and symbolic representations (García-García & Dolores-Flores, 2021; Oktaviyanti & Agus, 2023; Žakelj & Klančar, 2022). Cognitive limitations, such as low spatial ability and poor conceptual understanding, contribute to these issues (Liben, 2022; Uttal et al., 2024). Affective barriers like math anxiety and low self-confidence, as well as instructional methods that neglect visual strategies, also hinder visual thinking development (Hussein & Csíkos, 2023; Medina Herrera et al., 2024; Nilimaa, 2023). Consequently, many pre-service teachers struggle to teach mathematics visually, reducing their effectiveness in classroom instruction (Mainali, 2022; Weingarden & Heyd-Metzuyanim, 2024; Bernadetha Rizki Kaize et al., 2024).

Theoretical perspectives suggest that visual thinking is influenced by three interrelated domains: cognitive (Arnheim, 2020), affective (Reed, 2021), and environmental (Albert et al., 2022). From the cognitive domain, spatial reasoning and conceptual mastery are central to effective visual representation. Affective elements such as motivation, confidence, and anxiety affect students' willingness and persistence in using visual strategies. Meanwhile, the environmental context, particularly instructional strategies, media, and access to visualization tools, either supports or inhibits visual thinking development. These domains form the basis for the indicators used in the questionnaire and qualitative instruments in this study.

Although research on mathematical visual thinking has been conducted in various contexts, such as problem-solving, geometric reasoning, and the use of visualization technology, most of these studies emphasize cognitive dimensions alone (Del Cerro Velázquez & Méndez, 2021; Dorel, 2024; Downton & Livy, 2022; Sholahudin & Oktaviyanti, 2023). There is limited exploration of how affective and environmental factors also shape this skill, especially in teacher education contexts. Therefore, this study offers a novel contribution by investigating visual thinking through a holistic lens, integrating cognitive, affective, and environmental dimensions using mixed methods. It also aims to map the factors comprehensively and suggest targeted pedagogical implications for visual-based instruction.

This study aims to fill this gap by investigating the cognitive, affective, and environmental factors that influence the development of mathematical visual thinking among pre-service mathematics teachers. It applies a mixed-methods descriptive design to offer both breadth and depth of analysis. Therefore, this study is guided by the following research questions: (1) How is the profile of students' visual thinking ability in quadratic functions? (2) What are the key cognitive, affective, and environmental factors influencing their visual thinking? (3) What challenges do students face in applying visual thinking in solving quadratic function tasks? The objective of this study is to identify the dominant factors that influence mathematical visual thinking among pre-service mathematics teachers using a mixed-methods descriptive approach. The findings are expected to inform future instructional strategies, curriculum design, and media development to enhance visual thinking in mathematics education.

## Method

### Research Approach and Design

This study employed a mixed-methods approach using a sequential explanatory design, where quantitative analysis was conducted first to identify patterns in pre-service mathematics teachers' visual thinking (Clark & Ivankova, 2016; Creswell, W. John; Creswell, 2022). This was followed by qualitative analysis to explore their experiences, strategies, and the influencing factors in depth. The quantitative phase included a visual thinking test and a questionnaire assessing cognitive, affective, and environmental factors. The qualitative phase employed interviews and document analysis to explore students' visual thinking processes and common difficulties. Quadratic functions were chosen as the focus because they serve as a critical transition from linear to nonlinear mathematical concepts. Unlike linear functions, quadratic functions involve more complex visual features such as parabolas, vertex locations, axis of symmetry, and transformations. These characteristics demand a higher level of visual abstraction and strategic reasoning, making them a suitable context for exploring mathematical visual thinking (Arnheim, 2020; Reed, 2021; Sumanik et al., 2025).

The indicators of visual thinking ability and influencing factors used in this study were developed from a synthesis of prior research on mathematical visual thinking (Arnheim, 2020; Elsayed & Al-Najrani, 2021; Vale & Barbosa, 2023; Siregar & Amir, 2025), particularly those focusing on visual discrimination (VD), visual perception (VP), and visual analysis of shapes (VS). These indicators ensured theoretical alignment between instrument design and conceptual foundations. A sequential explanatory design was adopted as it allows the researcher to initially map out students' visual thinking abilities quantitatively, and then follow up with qualitative data to explain and deepen the understanding of the patterns found (Abu & Toyon, 2021; Thornberg et al., 2022).

### Research Participants

The population of this study included pre-service mathematics teachers who had completed at least one course involving mathematical visualization (e.g., Calculus). Using purposive sampling, 150 students participated in the quantitative phase. For the qualitative phase, 15 students were selected using maximum variation sampling based on their visual thinking scores, 5 students from each category: high, moderate, and low. This ensured a balanced representation of different ability levels and provided rich, varied insights into their experiences and challenges.

### Research Instruments

This study utilized four validated instruments based on mathematical visual thinking theory:

#### Mathematical Visual Thinking Test

This test assessed students' ability to represent, manipulate, and interpret quadratic functions visually, based on five key indicators:

**Table 1.** Indicators of the Mathematical Visual Thinking Test

| Indicator                     | Description                                                     | Visual Thinking Classification                          |
|-------------------------------|-----------------------------------------------------------------|---------------------------------------------------------|
| Visual Representation         | Drawing a graph of a quadratic function from its equation       | VD (Visual Discrimination)                              |
| Representation Manipulation   | Modifying the graph when equation coefficients change           | VP (Visual Perception)                                  |
| Representation Interpretation | Extracting information from the graph (vertex, roots, symmetry) | VS (Visual Analysis of Shapes)                          |
| Visual Transformation         | Understanding how transformations affect the graph              | VP (Visual Perception) & VS (Visual Analysis of Shapes) |

| Indicator              | Description                               | Visual Thinking Classification                              |
|------------------------|-------------------------------------------|-------------------------------------------------------------|
| Visual Problem Solving | Using graphs to solve real-world problems | VD (Visual Discrimination) & VS (Visual Analysis of Shapes) |

Table 1 presents the key indicators used to assess students' mathematical visual thinking ability, categorizing their skills into visual discrimination, perception, and shape analysis.

### Visual Thinking Factors Questionnaire

This questionnaire identified the cognitive, affective, and environmental factors influencing visual thinking using a 5-point Likert scale. Indicators are shown below:

**Table 2.** Indicators of the Visual Thinking Factors Questionnaire

| Factor        | Indicator             | Visual Thinking Classification                      | Sample Statement                                                                                  |
|---------------|-----------------------|-----------------------------------------------------|---------------------------------------------------------------------------------------------------|
| Cognitive     | Spatial ability       | VD (Visual Discrimination)                          | "I can visualize the shape of a quadratic function graph just by looking at the equation."        |
| Cognitive     | Concept understanding | VP (Visual Perception)                              | "I find it easier to understand quadratic functions through graphs than through equations."       |
| Cognitive     | Thinking strategies   | VS (Visual Analysis of Shapes)                      | "I often sketch a graph to help solve problems."                                                  |
| Affective     | Motivation            | VP (Visual Perception)                              | "I am more engaged when quadratic functions are taught with visuals."                             |
| Affective     | Anxiety               | VS (Visual Analysis of Shapes)                      | "I feel anxious when trying to interpret graph changes in quadratic functions."                   |
| Affective     | Self-confidence       | VD (Visual Discrimination)                          | "I am confident in drawing graphs of quadratic functions."                                        |
| Environmental | Teaching influence    | VP (Visual Perception)                              | "My lecturer frequently uses graphs to explain quadratic functions."                              |
| Environmental | Technology use        | VS (Visual Analysis of Shapes)                      | "I use applications such as GeoGebra, PhET Simulation, or Desmos to understand quadratic graphs." |
| Environmental | Learning media        | VD (Visual Discrimination) & VP (Visual Perception) | "Interactive visuals help me understand quadratic functions better."                              |

Table 2 outlines the factors assessed in the questionnaire, covering cognitive, affective, and environmental influences on students' visual thinking. The questionnaire employed a 5-point Likert scale ranging from "Strongly Disagree" to "Strongly Agree."

### Semi-Structured Interviews

Fifteen students were interviewed to explore their strategies, challenges, and tools used in understanding quadratic functions visually. Sample indicators:

**Table 3.** Interview Indicators

| Indicator                  | Sample Guiding Question                                                  | Visual Thinking Classification                              |
|----------------------------|--------------------------------------------------------------------------|-------------------------------------------------------------|
| Visual Thinking Experience | “How do you experience learning quadratic functions using graphs?”       | VP (Visual Perception)                                      |
| Visual Thinking Challenges | “Which parts of quadratic functions are hardest to understand visually?” | VS (Visual Analysis of Shapes)                              |
| Visual Thinking Strategies | “Do you use any specific visual strategies for problem solving?”         | VD (Visual Discrimination) & VS (Visual Analysis of Shapes) |
| Environmental Influence    | “How do lecturers or materials help you visualize quadratic functions?”  | VP (Visual Perception)                                      |
| Technology Utilization     | “Do you use GeoGebra or similar tools? How do they help?”                | VD (Visual Discrimination) & VS (Visual Analysis of Shapes) |

Table 3 provides an overview of the key themes explored in the interviews, emphasizing students' experiences, challenges, and strategies in visualizing quadratic functions.

### Document Analysis

Student work on quadratic graph tasks was analyzed to identify errors and visual reasoning strategies.

**Table 4.** Document Analysis Indicators

| Indicator                       | Description                                                          | Visual Thinking Classification |
|---------------------------------|----------------------------------------------------------------------|--------------------------------|
| Representation Accuracy         | Accuracy in drawing quadratic function graphs                        | VD (Visual Discrimination)     |
| Concept Consistency             | Consistency between graphs and function properties                   | VS (Visual Analysis of Shapes) |
| Readability & Clarity           | Labeling and interpretation clarity                                  | VD (Visual Discrimination)     |
| Connection with Algebraic Forms | Ability to link equations with graph features                        | VP (Visual Perception)         |
| Common Errors                   | Patterns of visual misunderstandings (e.g., transformations, vertex) | VS (Visual Analysis of Shapes) |

Table 4 summarizes the aspects evaluated in students' work, focusing on accuracy, conceptual consistency, clarity, and integration with algebraic representations.

### Data Collection Procedure

Data collection was conducted in two phases. The quantitative phase began with administering the visual thinking test and questionnaire to 150 students. The results were analyzed to categorize students into high, moderate, and low visual thinking levels. The qualitative phase then involved in-depth interviews with 15 selected students, analysis of their work, and observations of their problem-solving processes related to visualization tasks.

### Data Analysis Technique

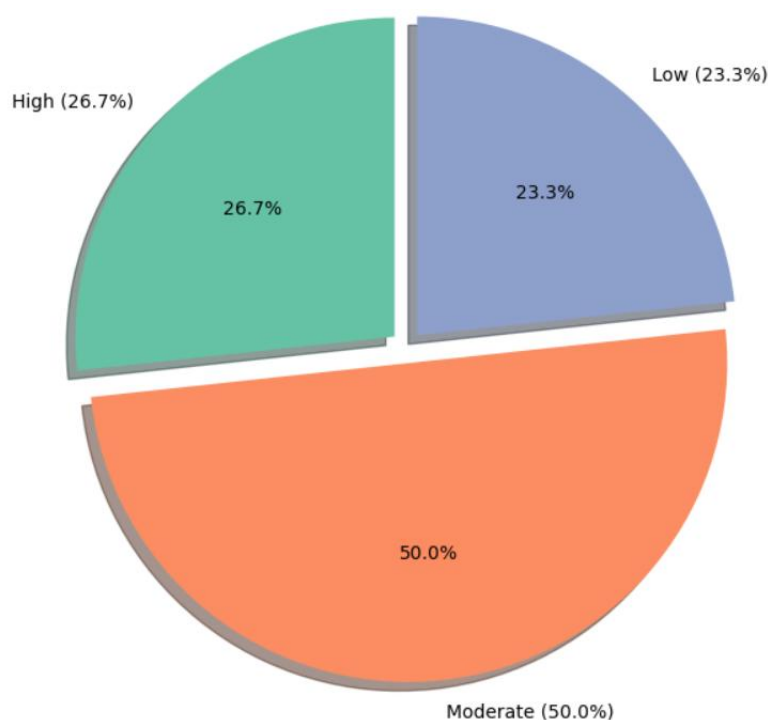
A triangulation strategy was employed to strengthen the validity of the findings (Flick, 2018; Swift, 2022). In the quantitative analysis, descriptive statistics were used to profile students' visual thinking, while Pearson correlation and multiple linear regression were applied to identify relationships among influencing factors and determine the dominant contributors. In the qualitative analysis, thematic coding was conducted on interview transcripts and student work (Lochmiller, 2021; Squires, 2023), allowing for the identification of recurring themes, which were

then compared and aligned with the quantitative patterns. The integration of both methods was carried out through a meta-inference process, where qualitative findings such as insights from students' interviews and documented problem-solving strategies were used to explain or contrast statistical trends from the quantitative phase. To ensure the trustworthiness of the qualitative findings, member checking and peer debriefing procedures were implemented. All quantitative data analyses and visualizations were performed using Python with the Matplotlib library (<https://python-fiddle.com/matplotlib>), ensuring reproducibility and transparency of the data-processing workflow.

## Results and Discussion

### Profile of Students' Visual Thinking Ability in Quadratic Functions

Visual thinking plays a crucial role in mathematics learning, especially in topics requiring graphical interpretation, such as quadratic functions. This study found that most students showed moderate levels of visual thinking. The distribution is visualized in Image 1 to enhance clarity.

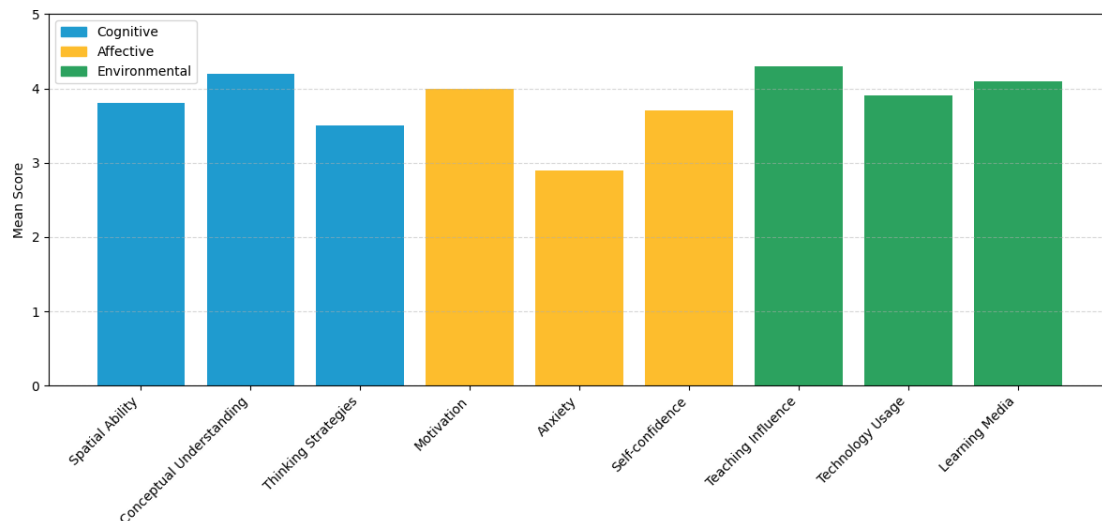


**Image 1.** Distribution of Students' Visual Thinking Ability Categories

Image 1 presents the distribution of students' visual thinking ability in quadratic functions, categorized into high, moderate, and low levels. Students in the high category demonstrated strong skills in sketching, interpreting, and transforming graphs, while students in the low group struggled with identifying key features like vertex and axis of symmetry (Bronkhorst et al., 2021; Liang & She, 2023). This study focuses specifically on visual thinking in the context of quadratic functions. However, the cognitive processes and visualization skills involved are also relevant to other mathematical topics such as trigonometry, calculus, and geometry, suggesting broader applicability for further research.

### Factors Influencing Students' Visual Thinking

To explore influencing factors, students responded to a questionnaire comprising cognitive, affective, and environmental aspects. Results are presented in Image 2.



**Image 2.** Mean Scores of Factors Influencing Visual Thinking

Image 2 summarizes the mean scores of various factors influencing students' visual thinking ability, categorized into cognitive, affective, and environmental aspects. The results indicate that teaching influence (4.3), conceptual understanding (4.2), and learning media (4.1) received the highest scores, highlighting the critical role of instructors and visualization in supporting students' comprehension of quadratic functions. This finding aligns with Yao et al. (2021), who asserted that effective use of visual representations in teaching enhances mathematical concept understanding. On the other hand, visual thinking anxiety scored the lowest (2.9), suggesting that most students do not experience significant fear when utilizing visual representations. This contrasts with Hussein and Csikos (2023), who argued that anxiety related to visual representations can be a major obstacle in mathematics learning. Among these, teaching influence, conceptual understanding, and learning media emerged as dominant. To optimize these factors (1) teaching influence, use dynamic visuals and iterative demonstrations; (2) conceptual understanding, strengthen algebraic-graphical linkage through targeted instruction, and (3) learning media, implement media that encourage active engagement (e.g., animation-based tools).

### Qualitative Insights: Strategies and Challenges in Visual Thinking

To complement the quantitative data, interviews and document analysis were used. Thematic coding revealed distinct patterns based on students' visual thinking levels.

**High-visual-thinking students** could easily visualize quadratic function graphs from equations, understand the effects of coefficient transformations, and use sketching strategies before solving problems. Sample interviews response: *"I find it easier to understand quadratic functions when I first sketch the graph before solving the algebraic equation."* (Respondent 3, high visual thinking).

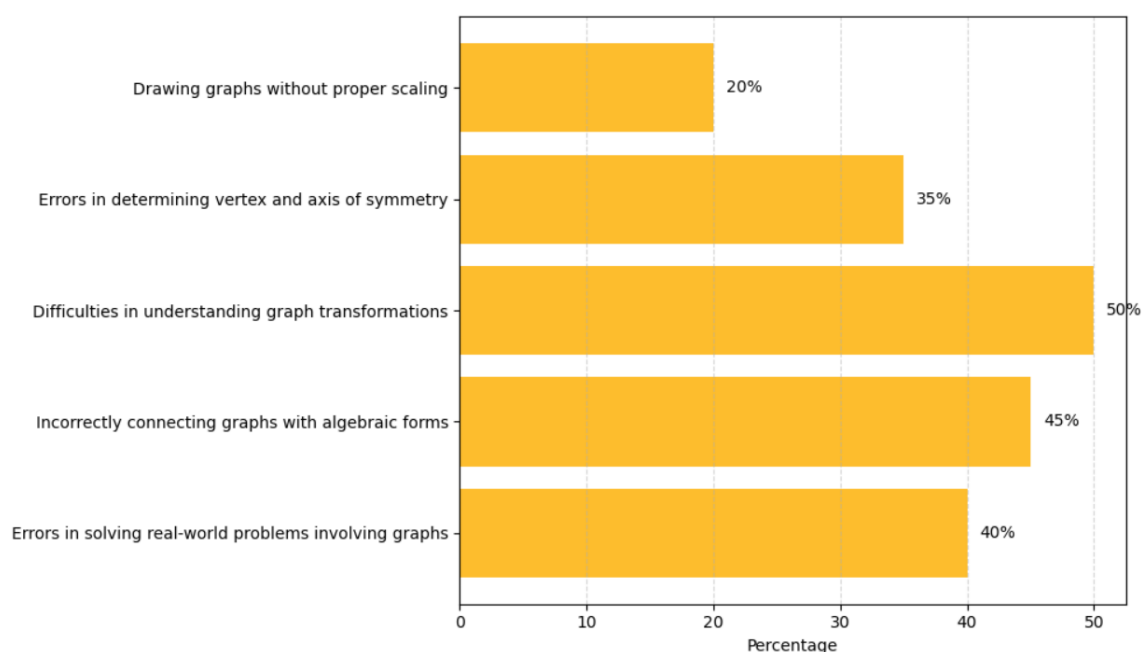
**Moderate-visual-thinking students** required assistive tools such as GeoGebra, PhET Simulation, or Desmos to comprehend graph transformations and struggled to connect equations with their corresponding graphs. Sample interviews response: *"I often use applications like PhET Simulation to ensure that my graph is drawn correctly."* (Respondent 9, moderate visual thinking).

**Low-visual-thinking students** relied heavily on algebraic procedures and faced difficulties in accurately sketching graphs, particularly in identifying the vertex and parameter changes' effects. Sample interviews response: *"I struggle to understand how coefficient changes affect the shape of the graph without seeing a pre-made example."* (Respondent 13, low visual thinking).

Qualitative findings help explain quantitative trends, offering deeper insight into students' reasoning and struggles. For example, those in the moderate group often relied on apps like GeoGebra, while high-ability students visualized graphs mentally and used sketching strategies. The integration between methods allowed explanation of *why* certain students performed better: those with better teaching experiences and motivation tended to show stronger visual reasoning across both the test and interviews. These findings support [Vale and Barbosa \(2023\)](#), who stated that individuals with high visual thinking tend to have more flexible strategies for visually representing mathematical information.

### Document Analysis: Common Errors in Visual Thinking of Quadratic Functions

Document analysis of student work identified frequent visual thinking errors (Image 3). The most prevalent issue was difficulty understanding graph transformations (50%).



**Image 3.** Frequency of Common Errors in Visual Thinking

Image 3 outlines common visual thinking errors observed in students' quadratic function tasks, with graph transformation difficulties being the most prevalent. The most significant error was understanding graph transformations (50%), indicating that students still struggle to grasp how coefficient changes influence graph shapes. This finding is consistent with [Gray and Holyoak \(2021\)](#), who argued that transitioning between algebraic and graphical representations is a major challenge in mathematical visual thinking. Furthermore, 35% of students struggled with determining the vertex and axis of symmetry, suggesting difficulties in connecting formulas with visual representations. [Žakelj and Klančar \(2022\)](#) emphasized that integrating symbolic and visual representations in learning is crucial for achieving deeper conceptual understanding.

The primary goal of this study was to identify the factors influencing mathematical visual thinking among prospective mathematics teacher students in the context of quadratic functions. The findings indicate that although the majority of students exhibit a moderate level of visual thinking, there is considerable variation in how they connect visual and algebraic representations in understanding the concept of quadratic functions. This discussion will further analyze the research findings by comparing them with previous studies and providing insights into the implications for mathematics teaching practice.

### **Level of Students' Visual Thinking Ability**

The results from the visual thinking test reveal that the majority of students fall into the moderate category (50%), with 23.3% in the low category and 26.7% in the high category. This indicates diversity in visual thinking abilities among the students, though none fully master visualization skills in the context of quadratic functions. These findings are consistent with [Mainali \(2021\)](#) study, which showed that while visual representation is critical in mathematics learning, many students struggle to connect mathematical concepts with corresponding images or graphs. Students in the low category demonstrated difficulties in accurately graphing, as well as in understanding the relationship between algebraic forms and graphs. This finding aligns with [Alam and Mohanty \(2024\)](#) assertion that mathematical visual thinking requires skills in linking various forms of representation, and that challenges in transitioning between algebraic and visual forms are a significant hurdle. The study also supports [Del Cerro Velázquez and Méndez \(2021\)](#) emphasis on the importance of spatial ability in visual thinking, as students with low spatial skills tend to struggle with visualizing changes in graphs when the coefficients in the function equations change.

### **Factors Affecting Students' Visual Thinking**

The survey results revealed that teaching methods and conceptual understanding are two primary factors contributing to students' visual thinking ability. Teaching methods scored the highest (4.3), indicating that instructional methods using visual representations to explain quadratic functions significantly influenced students' understanding. This supports [Medina Herrera et al. \(2024\)](#) findings, which suggest that teaching that emphasizes the visualization of mathematical concepts can effectively enhance student comprehension. Motivation (4.0) also received a high score, suggesting that motivated students are more likely to actively practice graphing and explore ways to understand the visual representation of quadratic functions. This finding is consistent with [Supriadi et al. \(2024\)](#); [Purnadewi & Widana, 2023](#) assertion that motivation plays a crucial role in developing visual thinking skills, as it encourages students to overcome difficulties encountered in learning mathematics. However, the anxiety factor was relatively low (2.9), indicating that most students did not experience significant anxiety or fear when dealing with visual representations. This may be due to students' prior familiarity with using visuals in learning contexts or the role of technology in reducing anxiety when graphing. This finding contrasts with [González-Gómez et al. \(2024\)](#) research, which suggests that anxiety toward visual representations can hinder mathematics learning, though anxiety did not appear to be a major factor in this study.

### **Use of Technology and Learning Media**

One significant finding of this research is the strong influence of learning media and the use of technology on students' visual thinking ability. Students who used software such as GeoGebra, PhET Simulation, or Desmos to graph functions demonstrated a better understanding of the relationship between algebraic functions and their graphs. This supports [Artasari et al. \(2024\)](#); [Widana & Laksitasari \(2023\)](#) assertion that technology can aid students in understanding difficult concepts, such as graph transformations in quadratic functions. However, the use of technology also revealed a dependency on visual aids. Students with moderate and low visual thinking abilities often relied on technology to verify results or to accurately graph functions. This suggests that while technology can enhance understanding, students who rely on these tools may not develop independent visualization skills, as highlighted by [Ng et al. \(2023\)](#). Therefore, the integration of technology in teaching should be balanced with independent practice to allow students to develop visual thinking skills without over-reliance on software.

### **Students' Difficulties in Visual Thinking of Quadratic Functions**

Document analysis showed that the greatest difficulties faced by students were in understanding graph transformations of quadratic functions (50%) and linking graphs to algebraic forms (45%). This suggests that although most students can graph quadratic functions correctly, they often

struggle to understand how changes in the coefficients of the quadratic equation affect the shape of the graph. These difficulties align with Ünal et al. (2023) assertion that understanding graph transformations requires the ability to connect algebraic and visual representations. Students often experience confusion when trying to comprehend how changes in the function's parameters affect the position or shape of the parabola. In this context, it is essential for mathematics instruction to emphasize the direct visualization of these changes and provide students with opportunities to draw and experiment with various coefficient values, as recommended by van Garderen et al. (2021).

### Practical Implications and Recommendations

Based on the findings, this study offers several practical implications for mathematics teaching, especially in the education of prospective mathematics teachers: 1) use of visual representations, educators should integrate manual graphing and mathematical software like GeoGebra, PhET, or Desmos to enhance students' understanding of quadratic functions through direct visual exploration; 2) focus on graph transformations, teaching should highlight the impact of parameter changes on quadratic function graphs by using iterative graphing techniques and step-by-step visual demonstrations; 3) enhancing motivation and reducing technology dependence, while technology aids learning, educators should encourage independent visual thinking through repeated practice to prevent over-reliance on digital tools; and 4) designing visual-based learning programs, integrating visualization with real-world applications in learning programs can enhance students' visual thinking and support their understanding of complex mathematical concepts.

### Linking to Artificial Intelligence and Future Implications

The development of students' visual thinking skills is increasingly relevant in the era of Artificial Intelligence (AI). AI-based educational tools (e.g., intelligent tutoring systems, adaptive graphing software) can support learners by offering instant feedback, personalized pathways, and visual cues based on learners' cognitive profiles (Gligorea et al., 2023; Halkiopoulos & Gkintoni, 2024). Integrating AI into visual thinking instruction may foster deeper conceptual understanding and offer scalable support for learners with different visual skill levels.

## Conclusion

This study aimed to investigate the levels of visual thinking skills, influencing factors, and common visual errors among prospective mathematics teachers in the context of quadratic functions. The findings revealed that most students possessed moderate visual thinking abilities. Key influencing factors included teaching strategies, conceptual understanding, motivation, and technological support. However, students often struggled with graph transformations and connecting algebraic forms to graphical representations. These results underscore the importance of instructional approaches that integrate visual tools with symbolic reasoning. To improve students' visual thinking, educators should design balanced learning experiences that emphasize both visualization and independent problem-solving.

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